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ORIGINAL ARTICLE

Applying embodied cognition: from useful interventions and their theoretical underpinnings to practical applications

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Abstract *Embodied trainings* allowing children to move their whole body in space have recently been shown to foster the acquisition of basic numerical competencies (e.g. magnitude understanding, addition performance). Following a brief summary of recent embodied training studies, we integrate the different results into a unified model framework to elucidate the working mechanisms of embodied trainings: Mapping processes, interaction between different regions of personal space, and the integration of different spatial frames of reference are addressed as potential factors underlying the effectiveness of embodied numerical trainings. In the concluding section, we elaborate on the practical applications of embodied numerical trainings in educational setting. We discuss under which circumstances embodied trainings work best, that is, for which age group and/or which numerical content embodied trainings should be most beneficial and which aspects need to be considered when aiming at applying embodied numerical trainings in formal educational settings like kindergartens or schools.

Keywords Embodied numerical trainings · Basic numerical skills · Numerical development · Number-space associations

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1 Introduction

Numbers are an integral part of everyday life: checking the time, comparing prices, estimating how much money you will get for your pension—all of these things are impossible without a basic understanding of number magnitudes and arithmetic operations. Interestingly, many of such daily encounters with numbers involve or imply bodily actions: for example, when asked how many nights to stay in a hotel while being in another city from Thursday to next Tuesday, most people will solve this by using their fingers as counting support. This simple example demonstrates how abstract concepts such as numbers are closely related to or even rooted in bodily actions (e.g. Moeller et al. 2012). This idea is reflected more generally in the concept of *embodied cognition* (e.g. Glenberg 2010) and was specified for numbers in the concept of *embodied numerosity* (Domahs et al. 2010; see also; Myachykov et al. 2013).

In the first part of this paper we will elaborate on this concept, outlining studies investigating bodily influences on cognition in general and numerical information in particular. Afterwards, we illustrate how specific whole-body movement was used to train basic numerical competencies.

2 Embodied cognition

The theoretical concept of *embodied cognition* makes specific predictions on how knowledge is acquired and represented in the human brain (e.g. Barsalou 2010; Glenberg 2010). The fundamental tenet of embodied cognition is that "[…] thinking is not something that is divorced from the body; instead thinking is an activity strongly influenced by the body and the brain interacting with the environment" (Glenberg et al. 2013, p. 573; see also; Glenberg 2010 for an overview). There are different approaches to embodiment (e.g. Barsalou 1999; Gallese and Lakoff 2005; Lakoff 1987) of which we will focus on the approach emphasizing the contribution of action to the shaping of cognition (cf. Glenberg 2010). In particular, we are interested in the contributions of motor activities on numerical cognition.

Specifying observations of links between motor activities and cognition, there is accumulating evidence linking the processing of numbers to motor activities. One of the most intuitive embodied interactions with numbers is finger use, for instance, in counting (e.g. Andres et al. 2012; Di Luca and Pesenti 2008). Processes and strategies involved in finger counting were repeatedly observed to influence symbolic numerical tasks such as magnitude comparison, addition or subtraction (e.g. Di Luca and Pesenti 2008; Domahs et al. 2010; Klein et al. 2011). For example, finger configurations (such as 'three' represented by an outstretched thumb, index and middle finger) presented as priming cues were found to influence subsequent magnitude comparisons: Participants responded faster and were less error prone when presented with cues that corresponded to their own counting habit and to the subsequently presented target (Di Luca and Pesenti 2008).

Movements of (other parts of) the body were also associated with number processing. Often, this replicated findings of a specific association of lateral movements in the left/right side of space with number processing (e.g. Hartmann et al. 2012; Loetscher et al. 2008; Shaki and Fischer 2014). The theoretical basis for this research is the mental number line—a conceptual metaphor describing a spatial representation of numerical magnitudes. According to this metaphor magnitudes are spatially represented along a horizontal line and (at least in Western countries) successively ordered from left to right (cf., Moyer and Landauer 1967; Restle 1970).

In accordance with the concept of such a mental number line, participants respond faster with their left hand to smaller magnitudes and with their right hand to larger magnitudes, even when number magnitude is not relevant for the task at hand (termed the Spatial-Numerical Association of Response Codes-, i.e. SNARC-effect, e.g. Dehaene et al. 1993). Important for embodied cognition research such number-space associations were also found to be influenced by bodily movements: for instance, head turns towards the left or right were observed to influence the generation of random numbers as participants generated more relatively smaller numbers when turning their head to the left as compared to turning it to the right (Loetscher et al. 2008). Recent studies also provide evidence for effects of whole-body movements (both active and passive) on number processing (e.g. Anelli et al. 2014; Hartmann et al. 2012; Shaki and Fischer 2014). For example, Shaki and Fischer (2014) showed that participants who generated random numbers while walking produced significantly smaller or larger numbers before they decided to turn to the left or right respectively. More importantly, determining turning direction beforehand had an according effect on the mean size of randomly generated numbers, suggesting bidirectional influences of numbers and space.

Within these lines of research involving an increasing range of movements—from fine-grained finger movements to gross whole-body movements—our embodied numerical trainings can be integrated. In a new approach to numerical learning, which involves task-specific, whole-body movement, we utilized the association of leftward movements with smaller numbers and of rightward movements with larger numbers to enhance children's basic numerical skills.

3 Embodied trainings

3.1 The embodied training concept

We define embodied trainings as trainings that allow for an embodied experience of a specific basic numerical concept (e.g. number magnitude). Most importantly, we therefore posit that the bodily movement should specifically match the content that is trained, rather than being unspecific. Regarding the trained content, we took into account the hierarchical structure of mathematic knowledge, which builds upon basic numerical competencies and basic arithmetic operations (Butterworth 2005; Jordan et al. 2009). Accordingly, our embodied trainings were designed to specifically address basic competencies, in particular children's spatial representation of number magnitude (see Dackermann et al. 2016a; Fischer et al. 2015a for overviews). There is accumulating evidence that spatial-numerical associations develop early in life (e.g. de Hevia et al. 2012, 2014) and become culturally shaped along with the practice of counting objects (e.g. Opfer and Thompson 2006), the experience of spatial movements (Patro et al. 2016), and reading/writing directions (e.g. Göbel et al. 2011) during childhood.

In our studies, we evaluated training success by assessing children's performance in a number line estimation task. Therein, children have to estimate the spatial position of numbers on an empty number line with labeled endpoints (e.g. between 0 and 100, cf. Siegler and Opfer 2003). For a long time, it was argued that performance in this task allows for direct inferences about the structure of the underlying spatial representation of number magnitude, also known as the mental number line (see Dackermann et al. 2015 for a more detailed discussion). However, considering recent findings (e.g. Barth and Paladino 2011; Cohen and Blanc-Goldhammer 2011; Dackermann et al. 2015; Link et al. 2014a), it rather seems that estimation performance is influenced by specific solution strategies: with increasing age children become more proficient at applying proportion-based strategies using start, mid-, and endpoint as reference points (Barth and Paladino 2011; see also; Peeters et al. 2016).

In sum, when training number line estimation or using it to evaluate training outcome, a resulting improvement does not necessarily indicate an improvement in children's mental number line representation. Instead, children's estimation performance might reflect their relational knowledge about numbers. They have to apply their knowledge about the relations among numerical magnitudes (e.g. 50 is halfway from 100, or 48 is smaller than 50) as well as their knowledge about the interval-scaled ordering of numbers along the number line (e.g. knowing 99 is located before 100). This knowledge may not reflect a spatial representation of number magnitude directly, but rather represents fluency with sequence values and magnitude relations.

Nevertheless, it is of particular importance to improve these skills, seeing that they reflect one of the most important steps in the development of numerical competencies (see Fritz et al. 2013). Thus, the design of our embodied trainings followed the spatial orientation of the mental number line (e.g. movements to the left for smaller numbers, movements to the right for larger numbers), aiming to train children's basic numerical competencies incorporating whole-body movement.

3.2 Embodied numerical training studies

In the following section, we briefly summarize our research. Complexity of tasks and movements was adapted to children's age and evolved from simple tasks such as magnitude comparison (Fischer et al. 2011) and number line estimation tasks (Fischer et al. 2015b; Link et al. 2013) towards more complex constructs such as place-value integration (Link et al. 2014b) and equidistant number spacing (Dackermann et al. 2016b).

When evaluating the training studies summarized in the following it is important to note that outcome measures varied across studies, in order to fit the training content adequately to the trained age group. The only task used as an outcome measure in all studies was a number line estimation task (see Sect. 3.1 for a description), as it closely reflects how numerical concepts were trained. Nevertheless, even this task was adapted with respect to the addressed number ranges according to children's age in the different training studies.

3.2.1 Study 1: Categorical movements on a dance mat

In the training study of Fischer et al. (2011), we developed and evaluated an embodied training of number magnitude comparison with categorical whole-body responses on a digital dance mat. Corresponding to the left-to-right orientation of the mental number line, kindergartners were trained to jump to the left/right field of the dance mat depending on whether a presented number was smaller/larger than a given standard (see Fig. 1). In the embodied training, the task was displayed along a number line to enhance training gain. The latter was not the case in the non-embodied control training, in which the to-be-compared numbers were displayed above each other on a tablet PC and rather than jumping to either side, children had only to indicate the larger number with an electronic pen (see Fig. 1).

Results indicated a clear benefit of the embodied training: within-subject comparisons revealed that children improved more strongly in number line estimation after the embodied training, compared to the control training. This differential benefit also generalized to children's understanding of counting principles, which were not trained at all. Results of a subsequent mediation analysis showed that this transfer effect was driven by children's improvement in number line estimation.

3.2.2 Study 2: Continuous movements along the number line

Given these promising results, we further extended the whole-body experience of numerical concepts from categorical (i.e. one step to the left or to the right) to continuous movements along a number line (Link et al. 2013). Children were trained on a number line estimation task requiring continuous movement along the number line. First-graders had to walk up to 3 m along a number line that was taped to the floor (ranging from 0 to 100) until they reached the estimated position of a target number (starting from either 0 or 100). Movements in 3D space were recorded by a Kinect sensor. Children started each trial at a point from which they had an overview of the whole number line (marked with an X in Fig. 1). In the control training, children had to perform the same number line estimation task on a tablet PC using a computer mouse. Starting position of the mouse pointer varied across items.

Results extended the findings of Fischer et al. (2011), because, even compared to a very strict control training, the embodied training led to comparable specific training effects in number line estimation. More importantly,

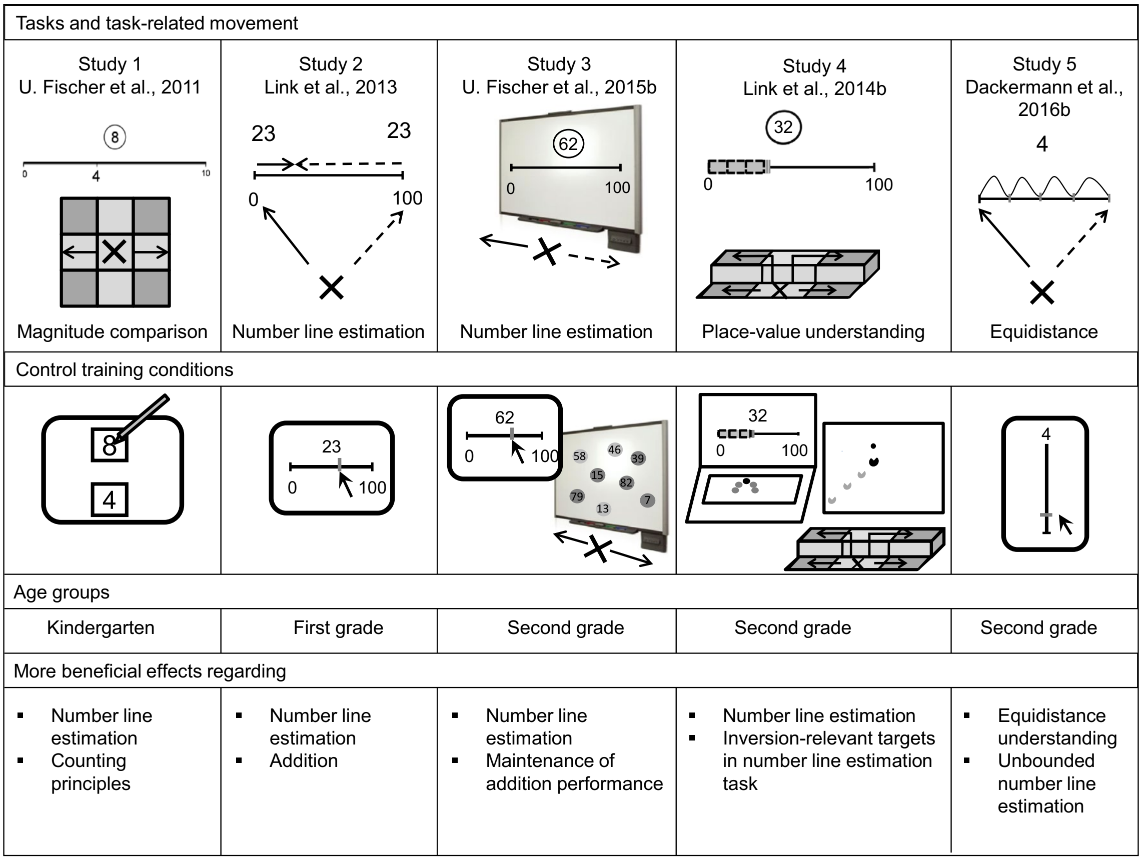

Fig. 1 Schematic illustration of embodied training studies. *Arrows* depict movement directions; *cross marks* indicate the starting point of the different training tasks. (Figure adapted from Dackermann et al. 2016a)

transfer effects to single-digit addition performance were only observed after the embodied training.

3.2.3 Study 3: Continuous movement along the number line using classroom media

In a subsequent pilot study, further evidence for the benefit of embodied training studies was provided using a medium that can already be found in numerous classrooms: an interactive whiteboard (Fischer et al. 2015b). Second-graders moved continuously along the whiteboard, also training number line estimation, with a shorter number line (1.5 vs. 3 m in Link et al. 2013). In the embodied training condition, children started from the position of 50 and walked along the whiteboard to locate their estimate on the 0–100 number line (see Fig. 1). Additionally, we implemented two control conditions: to control for the numerical content, children undertook the same numerical task on a tablet PC; and to control for motivational influences caused by the medium and children's movement, they performed a color discrimination task on the whiteboard with similar full-body movement but no numerical content.

Comparing three groups of children that were trained with the three conditions, results revealed the most pronounced improvement in number line estimation after the embodied training and the least benefit after the color discrimination training. Furthermore, we observed a motivational effect of the whiteboard medium in an addition task presented at a laptop: Only children that were trained on the whiteboard were able to maintain their motivation and concentration level to solve complex arithmetic problems after the training. Children trained in the PC condition rather showed a performance decrease after the training.

3.2.4 Study 4: Place-value integration on the number line

In the study by Link et al. (2014b), children had to estimate tens and units separately in a number line estimation task to train their understanding of the place-value concept. Placing the dance mat on a step, with the upper left and right fields requiring more physical effort to be stepped on, second-graders were trained with a different version of the number line estimation task: They were required to let a bar grow in length along the number line until it reached the estimated target position. Fields on the left side of the dance mat were associated with bar decrease, whereas fields on the right were associated with bar increase. Steps to the upper fields changed bar length by ±10, steps to the lower fields by ±1 (see Fig. 1). Again, two control training conditions were implemented: one trained the exact same content on a laptop whereas another one controlled for the potential motivational aspect of the dance mat (see Fig. 1). In the media-matched control task, children had to navigate a pacman horizontally on the screen so that it reached a displayed target point. Stepping on the upper (±10) or lower fields (±1) of the dance mat directed the pacman to either the left or right exactly as it let the bar grow in the condition with the number line estimation task.

Results revealed no difference between control conditions, but the embodied training yielded a more pronounced training effect on children's number line estimation performance. Moreover, a specific effect on children's place-value understanding was observed only after the embodied training. Here, children showed a significant improvement in their performance on target numbers for which an inversion error (confusing tens and units), leads to a specifically large estimation error (e.g. 18–81, cf., Helmreich et al. 2011).

3.2.5 *Study 5: Training the understanding of equidistant number line spacing*

Lifting the embodied training idea to a more abstract level, in one of our latest studies we trained children's understanding of the equidistant spacing of adjacent numbers on the number line (Dackermann et al. 2016b). Realizing this idea in an embodied training format, second-graders were trained to divide a line taped to the floor (1.5 or 2.5 m) into segments by walking along the line with a specific number of equally spaced steps. Their movements were recorded with a Kinect sensor, which also provided children with feedback videos comparing their estimated segmentation to the correct equidistant segmentation. Again, children started the task from a point from which they had an overview of the complete line (marked with an X in Fig. 1). The feedback video was displayed on a screen when children arrived back at the starting point after finishing a trial. In the control training, children performed the same task on a tablet PC using an electronic pen.

Specific training effects were evaluated using a paper and pencil version of the training task, whereas transfer effects were evaluated by bounded (both ends labelled, e.g. between 0 and 100) and unbounded number line estimation (only 0 and the unit 1 given; Cohen and Blanc-Goldhammer 2011; Link et al. 2014a) as well as addition and subtraction tasks.

At first sight, results were quite inconsistent: The embodied training led to a more pronounced specific training effect on children's equidistant spacing as well as a transfer effect to their unbounded number line estimation performance. In contrast, children showed improved performance only on bounded number line estimation and subtraction after the control training. However, these improvements were not significantly larger than those after the embodied training.

Taken together, all embodied training studies so far provided converging evidence in at least three aspects: (i) they led to specific training effects; (ii) this specific training effect was either comparable or more pronounced than in any of the control trainings; and (iii) transfer effects to not-directly-trained tasks were observed following the embodied training condition only (aside from the control training effects in Study 5; Dackermann et al. 2016b; see also; Looi et al. 2016 for positive effects of an embodied training in adults).

These positive results are even more remarkable considering that they were found although the trainings differed in several aspects: (i) the trained tasks (i.e., magnitude comparison, number line estimation, place-value understanding, and equidistant spacing), (ii) the expansion of incorporated movement (i.e., categorical movement on the digital dance mat or continuous movement along a number line), and (iii) the trained age group (i.e., kindergarteners, first- and second-graders). Thus, before suggesting a novel theoretical account integrating the mechanisms of embodied numerical trainings, we briefly summarize these qualitative differences between trainings to elaborate what might or might not be determining factors.

3.3 Qualitative differences of training formats

3.3.1 *Task-related movement*

Comparing our embodied numerical training studies, it is apparent that they involved different training tasks and, accordingly, different amounts of movement. In Study 1 (Fischer et al. 2011) we trained children with a number comparison task, using a digital dance mat that allowed only single steps to the left or right. Similar categorical movements were used in Study 4 (place-value training; Link et al. 2014b). In this study, four input fields of the dance mat were used to allow for differentiation between the input of tens and units.

In the other three studies, continuous movements along a physically presented (number) line were incorporated. We extended the physical space in which children were able to move using an interactive whiteboard (Study 4; Fischer et al. 2015b) or the Kinect sensor (Studies 2 and 5; Dackermann et al. 2016b; Link et al. 2013). While the input format on the interactive whiteboard mostly presented an extension of paper and pencil number line estimation tasks, the Kinect allowed for children's bodies to become input devices. Thus, children themselves became part of the task in either number line estimation (Study 2) or equidistance spacing (Study 5). Furthermore, in both studies we trained children to map numbers flexibly on to space by using different lengths of (number) lines.

3.3.2 Control training conditions

In Study 1 (Fischer et al. 2011) we varied both presentation *and* response format from the embodied to the control training condition: in the embodied training, the training task was displayed in a number line format, whereas in the control condition to-be-compared numbers were presented above each other (Fig. 1). Thus, more pronounced training effects after the embodied training condition might not be ascribed solely to differences in task-relevant bodily movement.

However, we took this methodologically important point into account in all following studies, allowing for a stronger interpretation of their results. In both Study 2 (Link et al. 2013) and Study 5 (Dackermann et al. 2016b), we trained children with the exact same numerical task in an embodied and a non-embodied training format. More importantly, we incorporated task-*irrelevant* movement in both conditions but task-*relevant* movement only in the embodied training condition. Even in comparison to these stricter control trainings both studies showed comparable or more pronounced specific training effects as well as more pronounced transfer training effects for the embodied trainings.

We even took this one step further in Study 3 (Fischer et al. 2015b) and Study 4 (Link et al. 2014b) by implementing two control trainings that did not only control for the numerical content trained, but also for the potential motivational effect of the media-supported training and the incorporated movement. Therefore, these studies allowed for the strongest interpretation: in both studies, the embodied training led to the most pronounced improvement, underlining the importance of combining numerical content and task-specific, whole-body movement to maximize training effects.

To summarize, neither the attractive medium nor the training task type per se could account for the differential training effects whenever these factors were controlled (as in Studies 3 and 4)—it seems to be the tailored combination that is essential.

3.3.3 Age groups

Another difference between studies is that different age groups were trained, ranging from kindergartners (Study 1) to first- and second-graders (Studies 2–5). In each study, the age group was chosen based not only on task difficulty but also on the developmental trajectory of the trained competencies. For example, while 4 year-olds already exhibit directional number-space associations when comparing non-symbolic numerosities (e.g. Patro and Haman 2012), children only start to develop number-space mappings for the number range 0–100 at the end of kindergarten and do not show reliable linear, interval-scaled mappings before grade two (e.g. Booth and Siegler 2008).

3.3.4 Underlying representations and transfer effects

Although all trainings were conceptualized based on the assumption of a left-to-right-oriented mental number line, they addressed different aspects of this representation (see Patro et al. 2014, for a more detailed taxonomy proposal on number-space associations). This, in turn, might have caused the different transfer effects we observed. By training number magnitude comparison, Study 1 (Fischer et al. 2011) addressed the directionality of the spatial magnitude representation: the association of small numbers with the left and large numbers with the right side of space. In this vein, it seems reasonable that children improved their performance in a counting task that also involves this directional aspect (e.g. Opfer and Thompson 2006).

In the three number line estimation trainings (Studies 2–4), a more elaborated number-to-space mapping was addressed. According to Patro et al. (2014), the number line estimation task does not evaluate a spontaneous but a systematic interval-scaled mapping of numbers to space. Therefore, training number line estimation should specifically corroborate the interval-scaled character of the linear representation of number magnitude compared to the mere mapping of numbers and space.

In the place-value training (Link et al. 2014b), the task not only required the correct identification of the digits and their respective position within target numbers, but also the correct activation of the digit's value in the tens' and the units' place (cf., Nuerk et al. 2015). This is indicated by the finding that children not only improved in number line estimation in general but particularly so for inversion-relevant target numbers.

The question of why children in Study 2 (Link et al. 2013) also improved their addition performance might be answered best by referring to processes like halving, quartering, magnitude comparison, simple arithmetic, etc., that are likely to be involved in elaborate number line estimation performance using proportion-based strategies (cf., Link et al. 2014c). Thus, training number line estimation might also train these processes implicitly (cf., Dackermann et al. 2015).

Study 5 (Dackermann et al. 2016b) specifically addressed the scaling aspect of the spatial magnitude representation. Transfer effects to bounded or unbounded number line estimation performance therefore seem plausible. However, since training conditions showed differential effects, the results require further theoretical elaboration (see Dackermann et al. 2016b, for a more in-depth discussion).

In sum, given these profound differences between studies, it is even more remarkable that all trainings showed comparable specific as well as transfer training effects. In the following section, we provide a theoretical framework of mechanisms that we suggest might capture the benefit of embodied numerical trainings.

4 A theoretical framework for embodied training effects

When children move their bodies in or through space while training numerical content we suggest that three different working mechanisms may be responsible for the advantage of embodied numerical training conditions. First, the necessary *mapping of number magnitude and physical space* (e.g. larger movements on the number line for larger numbers, as in Study 2); second, the *interaction between different regions of personal space* which refers to the embodied interaction of the learner with information presented at different distances from him/her (e.g. reacting by a step to the left or right to stimuli projected on to the floor approximately half a meter in front of oneself, as in Study 1); and third, the *integration of different spatial frames of reference*, referring to the flexible adaptation of different perspectives on the training task (e.g., when overlooking the entire number line when standing at 0 compared to parts of the number line behind and in front of oneself when actually walking on the number line, as in Study 2).

4.1 Potential mechanisms

4.1.1 Mapping physical space and number magnitude

The first working mechanism that may underlie embodied training effects might simply reflect that bodily movements in the training tasks allowed for a congruent mapping of movements in physical and number space—using the mental number line as a conceptual metaphor (Lakoff and Núñez 2000). This is obvious in Study 2 (Link et al. 2013), where children had to walk longer distances on the number line the larger the respective target number. Moreover, the same principle also applies to Study 4 (Link et al. 2014b), where more extensive physical movement was needed for the input of tens (on the step) than for the input of units (on the floor). Additionally, numerically larger numbers (requiring more physical movement) were located further towards the right on the number line. In accordance with what Bueti and Walsh (2009) termed *monotonic* or *intuitive mapping*, this led to the congruent mapping of one magnitude dimension to another (e.g. farther in physical space was associated with numerically larger quantities).

Importantly, Study 2 with the Kinect (Link et al. 2013) also included an incongruent mapping condition. As children started walking on the number line from either 0 or 100, larger numbers were not always associated with the physical experience of walking farther on the number line but with proportionally less physical movement when starting at 100. Similarly, in Study 3 with the whiteboard (Fischer et al. 2015b), the starting point was located at the position of number 50, so that numbers with the (absolute) largest numerical distance from 50 (i.e. 1 and 99) were associated with the largest amount of movement.

Another mapping mechanism possibly involved in the Kinect trainings (Studies 2 and 5, Dackermann et al. 2016b; Link et al. 2013) concerns different line lengths. Children in Study 2, for example, learned how to map magnitudes on to space not in a fixed (i.e. the position of 40 is always at the same distance from the starting point), but in a flexible way (i.e. the position of 40 needs to be scaled to the length of the number line). Because the line length in the embodied training varied from 1.5 to 3 m, they had to adapt the distance that they walked for each trial. These flexible mappings were not only incorporated in the embodied but also in the control trainings. Thus, fixed/flexible mapping mechanisms may contribute to embodied training effects but, since they were also part of the control trainings, they cannot sufficiently explain the working mechanisms.

Regarding these postulated congruent/incongruent and fixed/flexible mapping mechanisms, we argue that these processes build upon each other. A combination of two reliable mappings should corroborate task performance, as in the combination of congruent + fixed number-space mappings (e.g., always stepping the same distance to the right for large numbers on the dance mat in Studies 1 and 3, cf., Link et al. 2014b). For congruent + flexible as well as incongruent + fixed number-space mappings, it is necessary for participants to either adjust the distance for a different number line length (e.g. 1.5 or 3 m in Study 2) or to adjust for a different starting point (e.g. 0 or 100 in Study 2). In this way, children acquire relational knowledge not only with regard to number magnitudes but also to the relational scaling of physical space. The ability to use incongruent + flexible mapping processes seems to be most demanding, with children having to adjust flexibly to the starting point and the length of the number line. However, having been trained to map targets from fixed starting points to varying line lengths, or from different starting points to the same line length, should help children to scale distances to varying line lengths from different starting points.

To sum up, we suggest that such mapping mechanisms are beneficial for training effects because they allow for a physical experience of number magnitude and its relational nature through whole-body movement. In turn, this should strengthen relational numerical knowledge.

However, mapping physical space and number magnitude does not provide a sufficient explanation for embodied training effects in general, since simpler movements (i.e. one step left vs. one step right; Study 1) also led to more pronounced improvements. Furthermore, the differentiation of fixed/flexible mapping was also part of several control training conditions (Study 2 and 5). Therefore, it might be informative to inspect possible differences between embodied and control training conditions more thoroughly from a spatial cognition perspective: Whole-body movement, independent of its continuous or categorical character, may have involved different aspects of physical and representational space as opposed to a static training on a computer or tablet PC. In the following, we illustrate why the interaction between different regions of personal space as well as the integration of different spatial frames of reference might be seen as working mechanisms underlying the beneficial effects of embodied numerical trainings.

4.1.2 Interaction between different regions of personal space

When structuring the space within which a person acts, three regions have been differentiated: personal space (the body's own surface), peripersonal space (within an arm's reach), and extrapersonal space (space beyond an arm's reach; e.g. Halligan et al. 2003). There is accumulating evidence that these three regions of space are represented separately, as suggested by research on neglect patients (for an overview see Halligan et al. 2003). The most important distinction for our embodied trainings seems to be between peri- and extrapersonal space. Research on the development of perception and modulation of peri- and extrapersonal space suggests that successful integration of information from extrapersonal space develops later in life than for peripersonal space (cf., Gabbard et al. 2007).

In our studies, the differentiation of peri- and extrapersonal space was present only in the embodied but not in the non-embodied control conditions. For example, the dance mat was placed in children's peripersonal space, but the task itself was presented in their extrapersonal space (i.e. projected onto the floor approx. half a meter in front of them, Study 1). Thereby, task performance in extrapersonal space was accomplished through whole-body movement on the dance mat in peripersonal space. Hence, besides the explicit numerical training content, a dynamic interaction of peri- and extrapersonal space was implicitly enclosed in the embodied training, possibly strengthening the association of numbers with physical space.

In contrast to the embodied trainings, the control trainings involved either (i) the same interaction between peri- and extrapersonal space but no specific association of numbers and space (e.g. navigating the pacman by stepping on the dance mat, as in Study 4), or (ii) analog numerical content but no interaction between peri- and extrapersonal space (e.g. performing the same numerical task on a tablet PC not involving any training-specific physical movement, as in Study 2).

As the differentiation into peri- and extrapersonal space accounts only for the embodied trainings in which these were clearly distinguishable, it is harder to apply with trainings involving dynamically interacting spaces; this means, trainings in which children become part of the task, such as walking along the number line (Study 2, Link et al. 2013). Importantly, however, such training tasks led to varying perspectives of the participants on the task and thus required the integration of differing spatial frames of reference. This aspect is discussed in more detail in the following paragraph.

4.1.3 Integrating different spatial frames of reference

In our embodied trainings, the necessity of flexibly adapting one's own perspective on the training task, and thus integrating different spatial frames of reference, might provide another working mechanism underlying the beneficial effects of embodied trainings (see also Dackermann et al. 2016b).

Generally, research distinguished between egocentric and allocentric frames of reference (e.g. Burgess 2006; Halligan et al. 2003). Within an egocentric frame of reference, the spatial positions of objects are coded relative to a person's own perspective, whereas within an allocentric reference frame, they are coded independently of the person's own position but relative to other objects. Moreover, Keulen et al. (2002) suggested an additional action-based frame of reference for which the spatial coding of objects is determined by the starting position of the reacting effector (e.g. a specific hand, or in our case the whole body of the training participants). Following the argumentation of Fischer et al. (2007, p. 127) that "there have been several recent indications that visually guided action relies on allocentric information, in addition to egocentric location information", it seems plausible to use the differentiation of egocentric and allocentric frames of reference to explain embodied training mechanisms.

In the embodied training conditions children's spatial frames of reference changed when body movement altered their perspective on the task and on themselves within the task. In the two studies in which children became part of the training task (Study 2 and Study 5; Dackermann et al. 2016b; Link et al. 2013), children could overview the task setting from the starting point. Thus, children were able to perceive the spatial extension of the (number) line independently of their own position, within an allocentric frame of reference. Yet, when starting to move their bodies along the (number) line, children had to adapt their perspective and update their spatial position using an egocentric frame, and since they moved their bodies, also an action-based frame of reference. However, to correctly estimate either a target's position on the number line (Study 2, Link et al. 2013) or subsequent equidistant segments (Study 5, Dackermann et al. 2016b), they needed to reconsider the initial allocentric perspective, comparable to a mental scaffold, to correct for the bias following the new perspective (e.g. for the distance that already lay behind them). In contrast, the control condition of both studies involved only an allocentric frame of reference. Children always saw the whole (number) line in front of them during a training trial.

Hence, considering the seemingly inconsistent results of the equidistance training study, the allocentric perspective from the control training may have been specifically beneficial with regard to performing bounded number line estimations by applying proportion-based strategies (see also Dackermann et al. 2016b). Moreover, one might speculate that the control training also had a beneficial influence on subtraction because bounded number line estimation was found to be associated with simple arithmetic repeatedly (e.g. Link et al. 2014, involving subtraction). Cohen and Sarnecka (2014) argued for differential influences of mensuration skills on number line estimation: While subtraction and division are supposed to be applied when solving bounded number line estimation, addition and multiplication should be specifically involved in unbounded number line estimation. Accordingly, an improvement in bounded number line estimation might also lead to an improvement of respective mensuration skills.

In sum, when children became part of the embodied training task, task-related movements required the integration of all three frames of reference and, moreover, the flexible adjustment of children's perspectives on the task. Just as the interaction between peri- and extrapersonal space may lead to a more elaborate association of numbers and physical space, this might also hold for switching between and integrating different frames of reference and adapting flexible perspectives. Following this argument, a training of the same numerical content on a static computer should not provide similar training effects.

4.2 The model framework

Taken together, we propose the interaction of different regions of personal space (e.g. peri- and extrapersonal space), as well as the integration of different spatial frames of reference, as potential working mechanisms driving the beneficial effects of embodied trainings in strengthening the association between numbers and physical space (see Fig. 2). Because mapping mechanisms require at least a basic understanding of number magnitudes, especially for incongruent and flexible mapping processes, we suppose them to be additional rather than essential processes. However, it is important to note

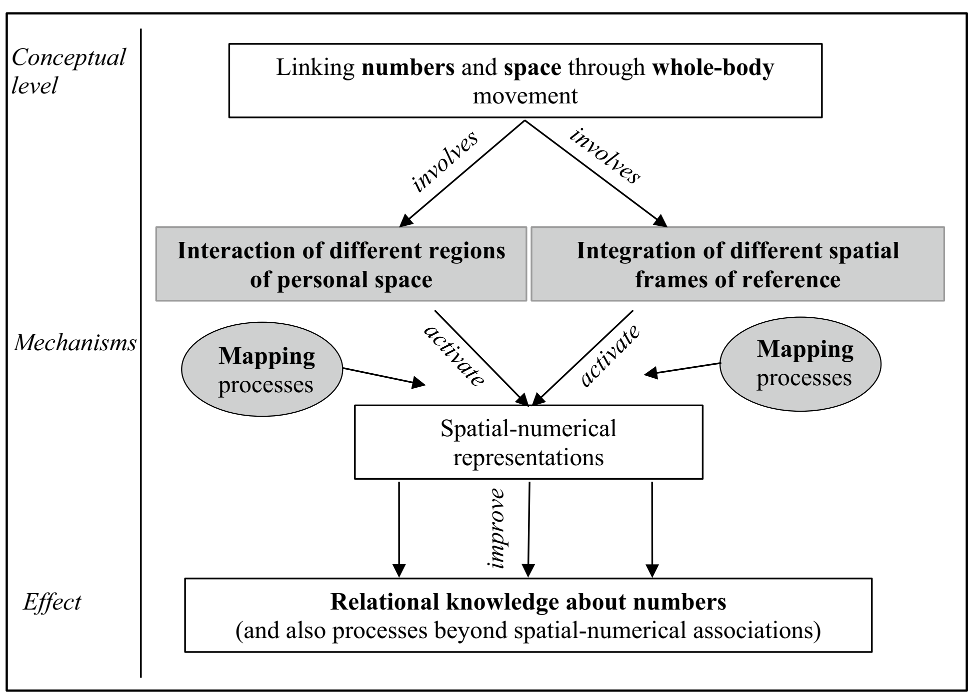

Fig. 2 Schematic illustration of the proposed mechanisms underlying embodied training effects. Linking numbers and space through bodily movements involves (i) the dynamic interaction of different regions of personal space (differentiating between peri-/extrapersonal, i.e. near and far, space) and (ii) the integration of different spatial frames of reference (differentiating between allocentric and egocentric spatial frames of reference). Additionally, we assume that spatial-numerical associations can be further strengthened by specific mapping processes

that, considered individually, none of the three proposed mechanisms provides a sufficient explanation for all of the embodied training effects observed so far. To decide which mechanisms are relevant for an individual training study, the training set-up and the involved regions of personal space in each task need to be considered carefully.

Regarding possible interactions amongst the postulated mechanisms, we suggest them to be interrelated via the spatial component: they either consider different regions of personal space, the integration of different spatial frames of reference, or the mapping of physical movement in space and number magnitude. Therefore, it seems likely that mapping processes are working mechanisms in addition to the interaction of peri-/extrapersonal space or egocentric/allocentric spatial frames of reference, presupposing the existence of a basic understanding of number magnitudes.

We assume that the two proposed working mechanisms, the interaction of different regions of personal space and the integration of different spatial frames of reference, which are involved when moving the body in physical space, promote spatial-numerical representations on a more general level. Because numbers are not only associated with a specific side of physical space (e.g. Dehaene et al. 1993), but movements in physical space (e.g. turning directions, Shaki and Fischer 2014) also show an impact on number processing, the association between numbers and space seems to be bidirectional.

As suggested previously (e.g. Bueti and Walsh 2009), numbers and physical space seem to share a common representational space. The demand to interact dynamically with different regions of personal space or integrate different spatial frames of reference by means of whole-body movement in embodied numerical trainings may corroborate these shared representations. Thus, the representational overlap of number and physical space may increase, resulting in a more thorough association. This might facilitate the activation of numerical representations and associated arithmetic processes.

When referring to the conceptual metaphor of the mental number line to explain this working mechanism, activation or even strengthening of the spatial component of the number line by interacting between peri- and extrapersonal space or flexibly adapting allocentric and egocentric perspectives, would make it easier to order and relate numbers to each other within and/or along space.

Although our model framework comprises mainly theoretical assumptions, practical conclusions can be drawn from both the model and the positive results of the embodied training studies. In the following section, we discuss suggestions for the future application of embodied trainings.

5 Practical applications

5.1 Suggestions derived from the model

As argued above, we are confident that embodied trainings are particularly efficient when they incorporate either a dynamic interaction of peri- and extrapersonal space or the flexible adaptation of ego- and allocentric perspectives on the learning content. While further research will be necessary, the role of the body in numerical learning, which has been discussed for a while now (e.g. Domahs et al. 2010), should find its way into practice.

Finger-counting has already been considered an important step in numerical development (e.g. Butterworth 2010; Fuson 1988), which may be considered in intervention. However, whole-body movement is—if at all—only part of mathematics instruction as a motivational component, but usually not linked specifically to any numerical content (in programs such as ‘Math and Movement’, https://mathandmovement.com). It would therefore be exciting to see whole-body movement that maps onto numerical representations while considering either an integration of peri- and extrapersonal space or varying perspectives integrated into intervention programs. Our model would predict that when integration of different regions of personal space or perspectives is required in the training (and successfully mastered by the child), learning success should be increased compared to trainings within only one region of personal space or with one and the same perspective on the training task. Future research will be necessary to test this prediction, but we are optimistic based on the results of our embodied trainings incorporating different perspectives (e.g. Study 4) or different spatial frames of reference (e.g., Study 2).

Additionally, acknowledging the general validity of embodied numerical trainings, there are new research questions to be addressed in future studies, allowing for a more differentiated knowledge of the working mechanisms. For instance, it would be important to evaluate the range of movement necessary to induce beneficial embodied effects in numerical trainings. In our trainings, children had to move their whole body (e.g. by stepping to the left or right or walking along a number line). Is this really necessary? Or might trainings involving smaller physical movements of only arms or legs be as efficient as those reported above? Virtual reality might even allow for dissociating actual physical movement from perceived movement on, for instance, a number line. Moreover, virtual reality might also allow for evaluating predictions derived from our model regarding the importance of integrating different perspectives on the learning material or interactions between peri- and extrapersonal space because these aspects can be manipulated easily.

However, apart from avenues for further research, there are also interesting aspects about applying embodied numerical trainings in educational settings.

5.2 Application in educational settings

As in all numerical trainings, the most obvious context for application would be a school setting. In our experience, teachers and students alike welcomed the training approach not only because of the expected training effects, but also because of the motivational appeal of 'jumping on the dance mat' or 'walking on the number line'. Yet, we also think that input from practitioners would help improve existing trainings, both with regard to the content, but also the design of the programs (see Fischer et al. 2015b for a discussion).

Implementation in actual classrooms would be difficult however, because only one child at a time would be able to train in the applied embodied settings. Yet, especially for children struggling with mathematics, research suggests a benefit of small group or individual training settings making it easier to appropriately address an individual student's needs (Kaufmann and Nuerk 2006; see also; Gersten et al. 2009 for a meta-analysis). Therefore, small group remediation classes might still be a possible setting for implementation of embodied trainings.

Another possibly limiting factor in an educational setting could be the media which need to be constantly available and set up for students to regularly perform the training. Also, a trainer would have to be present at all times to start the program, supervise the progress, and in some cases, provide feedback. Future research could of course go into the development of trainings that include automated feedback and do not require a trainer to deliver the training.

It is important to note though that embodied trainings do not necessarily require a computerized component when applied in educational settings. Taping a number line to the floor and walking along the line to practice number line estimation is only one possibility, in order to transfer embodied trainings to the classroom.

Such an application is already possible with the aid of some prevention programs for kindergarten children, such as the program 'Magnitudes, counting, numbers' (= 'Mengen, Zählen, Zahlen'; Krajewski et al. 2007). This program includes a 'number house', which is a linearly arranged mat made of fabric that could be laid on the floor, so that children can walk along the numbers. However, the number range in such prevention programs is rather limited (1–10) and would need to be extended for older children.

5.3 Application for different groups of children

In our research up to this point, we found that embodied numerical trainings were effective for children from kindergarten to second grade, and we propose that older children might also benefit from such trainings. However, we suggest that embodied numerical trainings should be particularly beneficial for younger children. Because spatial-numerical representations develop at an early age (e.g. Booth and Siegler 2008), an embodied experience of spatial-numerical associations should have the greatest impact while this development is still underway. The same may also be true for older children with mathematical learning difficulties, who often show deficits in basic numerical competencies (e.g. Butterworth 2010). A first, as of yet unpublished, study, in which we applied the embodied training with the dance mat to children with mathematical learning difficulties, lends support to this assumption. Accordingly, we are optimistic that the embodied training concept can be beneficial for children with different levels of mathematical abilities.

6 Conclusions

In sum, there is accumulating evidence that the success of embodied trainings can be generalized across different age groups (from kindergarten to second grade), different digital media (dance mat, interactive whiteboard, and Kinect sensor), and different numerical training contents (magnitude comparison, number line estimation, and equidistant spacing). To account for the beneficial effects of embodied numerical trainings, we proposed a model framework specifying the relevant working mechanisms: (i) mapping processes between numbers and physical space, (ii) the interaction between different regions of personal space, and (iii) the integration of different spatial frames of reference. When these mechanisms are taken into account, we are confident that with a match between children's abilities and the respective training task, embodied numerical trainings should be generally beneficial.

We conclude that embodied numerical trainings provide an innovative and valuable method for training basic numerical competencies in children of different skill levels. However, after the benefits of this method are established, the next step in our view should be integrating such methods into mathematical learning in- and outside school—accomplished in an interdisciplinary approach together by educators, educational scientists and mathematics didacticians, teachers, developmental psychologists, computer scientists as well as parents and children.

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