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SPECIAL SECTION: THE DEVELOPMENT OF MATHEMATICAL COGNITION

Playing linear numerical board games promotes low-income children's numerical development

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Abstract

The numerical knowledge of children from low-income backgrounds trails behind that of peers from middle-income backgrounds even before the children enter school. This gap may reflect differing prior experience with informal numerical activities, such as numerical board games. Experiment 1 indicated that the numerical magnitude knowledge of preschoolers from low-income families lagged behind that of peers from more affluent backgrounds. Experiment 2 indicated that playing a simple numerical board game for four 15-minute sessions eliminated the differences in numerical estimation proficiency. Playing games that substituted colors for numbers did not have this effect. Thus, playing numerical board games offers an inexpensive means for reducing the gap in numerical knowledge that separates less and more affluent children when they begin school.

Introduction

The mathematical understanding of children from low-income families trails far behind that of peers from middle-income families (Geary, 1994; National Assessment of Educational Progress, 2004). This discrepancy begins before children enter school. Preschoolers from impoverished backgrounds count, add, subtract, and compare magnitudes less well than more advantaged peers (Arnold, Fisher, Doctoroff & Dobbs, 2002; Jordan, Kaplan, Olah & Locuniak, 2006; Jordan, Levine & Huttenlocher, 1994). The early differences are related to later ones; a meta-analysis of six large longitudinal studies indicated that low-income kindergartners' mathematical knowledge is a strong predictor of their math achievement at ages 8, 10, and 13/14 years, a much better predictor than the kindergartners' reading, attentional capacity, or socio-emotional functioning are of those skills at the later ages (Duncan, Dowsett, Claessens, Magnuson, Huston, Klebanov, Pagani, Feinstein, Engel, Brooks-Gunn, Sexton, Duckworth & Japel, 2007).

The present study tests two hypotheses regarding the discrepancy between the mathematical knowledge of preschoolers from lower- and higher-income backgrounds. One is that low-income preschoolers' relatively poor mathematical performance reflects less frequent use of linear representations of numerical magnitude in situations that call for such representations. The other is that playing numerical board games can stimulate greater use of appropriate representations and thus improve low-income children's numerical competence.

Underlying both hypotheses are previous findings that people represent the magnitudes of numbers in multiple ways. Within the logarithmic ruler representation (Dehaene, 1997), mean estimated magnitude is a logarithmic function of actual magnitude (i.e. the distance between the magnitudes of small numbers is exaggerated and the distance between the magnitudes of larger numbers is understated). In contrast, within the accumulator (Brannon, Wusthoff, Gallistel & Gibbon, 2001) and linear ruler representations (Case & Okamoto, 1996), mean estimated magnitude is a linear function of actual magnitude. Within the accumulator representation, variability of estimates increases linearly with actual magnitude; within the logarithmic and linear ruler representations, variability and magnitude are unrelated. Frequency of use of the representations changes with age, and individuals often use different representations on similar tasks (Siegler & Opfer, 2003).

Increasing reliance on linear representations seems to play a central role in the development of numerical knowledge. Consider data on number line estimation. On this task, children are presented a series of lines with a number at each end (e.g. 0 and 1000), a third number (e.g. 37) above the line, and no other markings. The task is to estimate the location on the line of the third number. An advantage of this task is that it transparently reflects the ratio characteristics of the formal number system.

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Just as 80 is twice as great as 40, the estimated location of 80 should be twice as far from 0 as the estimated location of 40. More generally, estimated magnitude on the number line should increase linearly with actual magnitude, with a slope of 1.00.

Studies of number line estimation indicate that children's estimates often do not increase linearly with numerical size. On 0–100 number lines, kindergartners consistently produce estimates consistent with the logarithmic ruler representation (e.g. the estimated position of 15 is around the actual location of 60 on the number line). In contrast, second graders produce estimates consistent with the linear ruler model (Siegler & Booth, 2004). A parallel change occurs on 0–1000 number lines between second and fourth grades, with second graders' estimates fitting the logarithmic ruler model and fourth graders' fitting the linear ruler model (Booth & Siegler, 2006).

This age-related change in number line estimation is not an isolated phenomenon. Children undergo parallel changes from logarithmic to linear representations at the same ages on numerosity estimation (generating approximately *N* dots on a computer screen) and measurement estimation (drawing a line of approximately *N* units) (Booth & Siegler, 2006). Consistent individual differences are also present on these tasks, with children in the same grade usually producing the same pattern of estimates (e.g. linear) across the three tasks. Linearity of number line estimates also correlates with speed of magnitude comparison (Laski & Siegler, 2007), learning of answers to unfamiliar addition problems (Booth & Siegler, 2008), and overall math achievement test scores (Booth & Siegler, 2006).

Good theoretical grounds exist for these consistent relations between the linearity of numerical magnitude estimates and overall math achievement. Magnitudes are central within numerical representations; indeed, by second or third grade, children are unable to inhibit activation of the magnitudes of numbers even when the activation interferes with task performance (Berch, Foley, Hill & McDonough-Ryan, 1999; Nuerk, Kaufmann, Zoppoth & Willmes, 2004). Magnitude representations also influence arithmetic performance and accuracy of estimation of answers to arithmetic problems (Ashcraft, 1992; Case & Sowder, 1990; Dowker, 1997; LeFevre, Greenham & Waheed, 1993). When people speak of 'number sense', they are usually referring to ability to judge the plausibility of answers to numerical problems on the basis of the numbers' magnitudes (Crites, 1992; Siegel, Goldsmith & Madson, 1982).

The numerical experiences that lead children to form linear representations are unknown. It seems likely that counting experience contributes, but such experience appears insufficient to create linear representations of numerical magnitudes (Schaeffer, Eggleston & Scott, 1974).

If counting experience is insufficient to yield linear magnitude representations, what numerical experiences might contribute? One common activity that seems ideally designed for producing linear representations is playing numerical board games, that is, board games with consecutively numbered, linearly arranged, equal-size spaces, such as *Chutes and Ladders*. As noted by Siegler and Booth (2004), such board games provide multiple cues to both the order of numbers and the numbers' magnitudes. When a child moves a token in such a game, the greater the number that the token reaches, the greater: (a) the distance that the child has moved the token, (b) the number of discrete moves the child has made, (c) the number of number names the child has spoken and heard, and (d) the amount of time the moves have taken. Thus, such board games provide a physical realization of the linear ruler or mental number line, hypothesized by Case and Griffin (1990) to be the central conceptual structure underlying early numerical understanding.

For many young children, everyday informal activities, including board games, provide rich experiences with numbers, which seem likely to help them form linear representations. In general, children from higher SES backgrounds have greater opportunities at home to engage in such activities than children from lower SES backgrounds (Case & Griffin, 1990; Tudge & Doucet, 2004). These differences in experience with board games and other informal number-related activities seem likely to contribute to SES-related differences in early numerical understanding.

The intuition that playing board games might promote early numerical development is not a new one. The curricula Project Rightstart, Number Worlds, and Big Math for Little Kids reflect the same intuition (Greenes, Ginsburg & Balfanz, 2004; Griffin, 2004). However, those curricula and others designed to bolster low-income preschoolers' numerical understanding (e.g. Arnold *et al.*, 2002; Starkey, Klein & Wakeley, 2004; Young-Loveridge, 2004) include a variety of activities (counting, arithmetic, number comparison, board games, one–one correspondence games, etc.), thus precluding specification of the contribution of any one component.

The present study was designed to test whether playing a numerical board game in and of itself would enhance low-income children's knowledge of numerical magnitudes. We were particularly interested in whether such experience would help the children generate an initial linear representation of numerical magnitude, as indexed by their number line estimates. Therefore, we presented 4-year-olds from low-income backgrounds with four 15-minute sessions over a 2-week period. Children in the numerical game group played a linear board game with squares labeled 1–10; children in the color game group were given identical experience except that the squares in their board game included different colors rather than different numbers.

We therefore conducted two experiments. Experiment 1 examined the initial numerical estimates of low-income and middle-income preschoolers. Experiment 2 examined the effects of playing linearly arranged, numerical board games on low-income children's representations of numerical magnitude.

Experiment 1

Method

Participants

Participants were 58 preschoolers (30 males, 28 females), ranging in age from 4.0 to 5.1 years (*M* = 4.7 years, *SD* = .34). Thirty-six participants (*M* = 4.6 years, *SD* = .43, 56% female, 58% African American, 42% Caucasian) were recruited from a Head Start program and three childcare centers, all of which served low-income urban families. Almost all (96%) of these children's families received government subsidies for childcare expenses. The other 22 children (*M* = 4.8 years, *SD* = .30, 45% female, 77% Caucasian, 23% Asian) were recruited from a predominately upper-middle-class university-run preschool. The experimenters were a female graduate student of Indian descent and a female, Caucasian research assistant.

Materials

Stimuli for the number line estimation task were 20 sheets of paper, each with a 25-cm line arranged horizontally across the page, with '0' just below the left end of the line, and '10' just below the right end. A number from 1 to 10 inclusive was printed approximately 2 cm above the center of the line, with each number printed on two of the 20 sheets.

IV: income level
DV: magnitude and order knowledge

Procedure

Children met individually with an experimenter who told them that they would be playing a game in which they needed to mark the location of a number on a line. On each trial, after asking the child to identify the number at the top of the page (and helping if needed), the experimenter asked, 'If this is where 0 goes (pointing) and this is where 10 goes (pointing), where does *N* go?' The numbers from 1 to 10 inclusive were presented twice in random order, with all numbers presented once before any number was presented twice. No feedback was given, only general praise and encouragement.

Results and discussion

The ideal function relating actual and estimated magnitudes would be perfectly linear (R^2_{lin} = 1.00) with a slope of 1.00. That would mean that estimated magnitudes rose in 1:1 proportion with actual magnitudes. Linearity and slope are conceptually, and to some degree empirically, independent. Estimates can increase linearly with a slope much less than 1.00, and the best fitting linear function can have a slope of 1.00 but with more or less variability around the function. Therefore, linearity and slope were analyzed separately.

We computed the median estimate for each number of children from middle-income backgrounds, compared the fit of the best fitting linear function to those medians, and did the same for children from low-income backgrounds. The best fitting linear function for preschoolers from middle-income backgrounds fit their median estimates considerably better than did the best fitting linear function for peers from low-income backgrounds, R^2_{lin} = .94 versus .66. Analyses of individual children's estimates provided converging evidence. The best fitting linear function accounted for a mean of 60% of the variance in the estimates of individual children from middle-income backgrounds, but only 15% of the variance among children from low-income backgrounds, $t(56)$ = 5.38, p < .001, d = 1.49.

Analyses of the slopes of the best fitting linear function yielded a similar picture. The slopes of the best fitting linear function to the median estimates of middle-income and lower-income preschoolers were .98 and .24, respectively. Again, analyses of individual children's estimates provided converging evidence. The mean slope for individual children from higher-income families was higher (and closer to the ideal slope of 1.00) than that for children from lower-income families (mean slopes = .70 and .26, $t(56)$ = 4.28, p < .001, d = 1.16).

To examine knowledge of the order of numbers of children from higher- and lower-income backgrounds, we compared each child's estimate of the magnitude of each number with the child's estimate for each of the other numbers, and calculated the percent estimates that were correctly ordered. Children from higher-income backgrounds correctly ordered more of their estimates than did children from lower-income backgrounds, 81% versus 61%, $t(56)$ = 4.73, p < .001, d = 1.28.

Thus, as predicted, the estimates of preschoolers from low-income families revealed much poorer understanding of numerical magnitudes than did the estimates of preschoolers from higher-income families. Estimates of many children from low-income backgrounds did not even reveal knowledge of the ordering of the numbers' magnitudes; 60% of children correctly ordered fewer than 60% of the magnitudes of these single digit numbers.

Experiment 2

Experiment 2 tested the hypothesis that an hour of experience playing a simple, linearly organized numerical board game can enhance low-income children's knowledge of numerical magnitudes. Children from low-income backgrounds were randomly assigned to play one of two board games. The games differed only in whether each space included a number or a color, and whether children cited the number or color when moving their token. We predicted that playing the number-based board game would improve children's estimation and their representation of numerical magnitude relative to: (1) their numerical representation before playing the game, and (2) the representations of children who played the color-based game.

IV: number or color game
DV: numerical representation

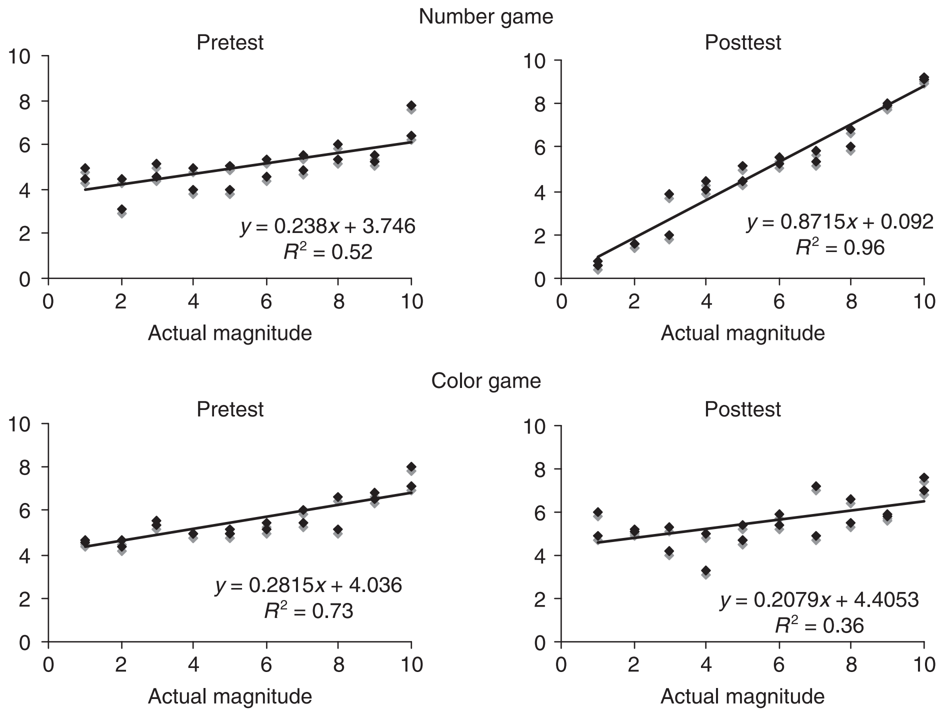

Figure 1 *Best fitting pretest and posttest linear functions among children who played the two board games.*

Method

Participants

Participants were the 36 children from low-income backgrounds in Experiment 1. Eighteen children were randomly assigned to the experimental condition (M = 4.6 years, SD = .30, 56% female, 56% African-American, 44% Caucasian). The other 18 children were randomly assigned to the control condition (M = 4.7 years, SD = .42, 56% female, 61% African-American, 39% Caucasian). The experimenter was a female graduate student of Indian descent.

Materials

Both board games were 50 cm long and 30 cm high; had 'The Great Race' written across the top; and included 11 horizontally arranged, different colored squares of equal sizes with the leftmost square labeled 'Start'. The numerical board had the numbers 1–10 in the rightmost 10 squares; the color board had different colors in those squares. Children chose a rabbit or a bear token, and on each trial spun a spinner to determine whether the token would move one or two spaces. The number condition spinner had a '1' half and a '2' half; the color condition spinner had colors that matched the colors of the squares on the board.

Procedure

Children met one-on-one with an experimenter for four 15-minute sessions within a 2-week period. Before each session, the experimenter told the child that they would take turns spinning the spinner, would move the token the number of spaces indicated on the spinner, and that whoever reached the end first would win. The experimenter also said that the child should say the numbers (colors) on the squares through which they moved their tokens. Thus, children in the numerical-board group who were on a 3 and spun a 2 would say, '4, 5' as they moved. Children in the color-board group who spun a 'blue' would say 'red, blue'. If a child erred, the experimenter correctly named the numbers or colors in the squares and then had the child repeat the numbers or colors while moving the token. Each game lasted 2–4 minutes; children played approximately 30 games over the four sessions. At the beginning of the first session and at the end of the fourth session, the experimenter administered the number line task, which served as the pretest and posttest.

Results and discussion

The number line estimates of children who played the numerical board game became dramatically more linear from pretest to posttest (Figure 1). On the pretest, the best fitting linear function accounted for 52% of the variance in the median estimates for each number; on the posttest, the best fitting linear function accounted for 96% of the variance. In contrast, the median estimates of children who played the color board game did not become more linear from pretest to posttest. The best fitting linear function accounted for 73% of the variance in the median estimates on the pretest, versus 36% of the variance on the posttest. Viewed from another perspective,

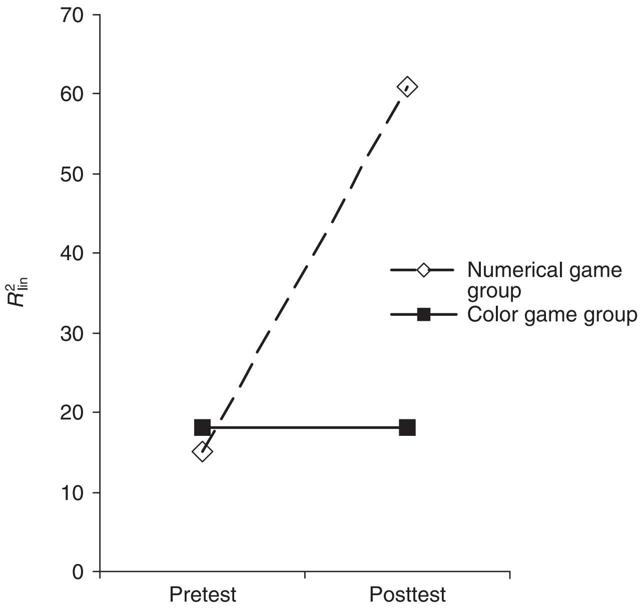

Figure 2 *Mean percent variance in individual children's pretest and posttest estimates accounted for by the linear function.*

the two conditions did not differ in the fit of the best fitting function on the pretest, but on the posttest, the best fitting linear function of children who played the numerical board game fit considerably better.

Analysis of the linearity of individual children's estimates provided converging evidence. Among children who played the numerical board game, the percent variance accounted for by the linear function increased from a mean of 15% on the pretest to a mean of 61% on the posttest, $t(17) = 6.92$, $p < .001$, $d = 1.80$ (Figure 2). In contrast, there was no pretest-posttest change among children who played the color version of the game; the linear function accounted for a mean of 18% of the variance on both the pretest and the posttest. The two groups did not differ on the pretest, but on the posttest, the estimates of children who played the numerical board game were much more linear, mean $R^2_{lin} = .61$ and .18, $t(34) = 4.85$, $p < .001$, $d = 1.62$.

To provide a more intuitive sense of the number line estimation data, the accuracy of children's estimates on the number line was calculated using the formula that *percent absolute error* equals:

$$\frac{\text{Estimate} - \text{Estimated Quantity}}{\text{Scale of Estimates}} \tag{1}$$

For example, if a child was asked to estimate the location of 5 on a 0–10 number line and placed the mark at the location that corresponded to 9, the percent absolute error would be 40% [(9–5)/10]. Thus, lower absolute error indicates more accurate estimates.

The accuracy data paralleled the linearity data. Among children who played the numerical board game, percent absolute error decreased from pretest to posttest (28% vs. 20%, $t(17) = 2.43$, $p < .05$, $d = .71$). In contrast, the absolute error of estimates among children who played the color board game did not change (27% vs. 28%).

Findings on the slope of estimates provided converging evidence for the conclusion that playing the numerical board game enhanced knowledge of numerical magnitudes. For the group median estimates, the slope generated by children in the numerical board game condition increased from pretest (.24) to posttest (.87). In contrast, the slope generated by children in the color board game condition did not change from pretest (.28) to posttest (.21). Analyses of individual children's slopes again provided converging evidence. The mean slope for children who had played the numerical board game increased from pretest to posttest (.23 vs. .71, $t(17) = 5.20$; $p < .001$, $d = 1.66$). In contrast, the mean slope for children who had played the color game did not change (.30 vs. .24). Again, no differences were present on the pretest, but the posttest slopes of those who played the numerical board game were higher (.71 versus .24, $t(34) = 4.00$, $p < .001$).

Finally, we examined pretest–posttest changes in knowledge of the ordering of the numerical magnitudes by comparing the estimated magnitudes of all pairs of numbers and computing the percentage of pairs for which the larger number's magnitude was estimated to be greater. Children who had played the numerical board game ordered correctly the magnitudes of more numbers on the posttest than on the pretest (81% vs. 62%, $t(17) = 5.57$, $p < .001$). Children who had played the color version of the game showed no pretest–posttest change (61% vs. 62% correctly ordered pairs).

Did children from lower SES backgrounds who played the board game 'catch-up' to age peers from higher SES backgrounds who participated in Experiment 1? The answer was 'yes': The posttest magnitude estimates of the low-income children who had played the numerical board game were equivalent on all measures to those of the middle-income children in Experiment 1. Consider the analyses of individual children's performance. There was no difference in the mean fit of the linear function to individual children's estimates (mean $R^2_{lin} = .61$ and .60). There was no difference in the mean slope of individual children's estimates (mean slopes = .65 and .66). There was no difference in percent correctly ordered pairs (81% in both cases). Thus, playing numerical board games with an adult for four 15-minute sessions over a 2-week period overcame the low-income 4-year-olds' initial disadvantage on all three measures of numerical knowledge and rendered their performance indistinguishable from that of middle-income peers.

General discussion

Findings from both experiments supported the hypotheses that motivated the study. Results of Experiment 1 indicated that 4-year-olds from impoverished backgrounds have much poorer knowledge of numerical magnitudes than age-peers from more affluent backgrounds. Results of Experiment 2 indicated that providing children from low-income backgrounds with an hour of experience playing board games with consecutively numbered, linearly arranged, equal-size squares improved their knowledge of numerical magnitudes to the point where it was indistinguishable from that of children from upper-middle-income backgrounds who did not play the games. Playing otherwise identical non-numerical board games did not have this effect.

The results also suggest a partial answer to a more general theoretical question: Why does the numerical knowledge of low-income preschoolers lag so far behind that of middle-income peers? Children with greater exposure to numerical board games enter kindergarten with greater intuitive knowledge of numbers (Case & Griffin, 1990; Phillips & Crowell, 1994). Although designed to promote enjoyable parent–child and child–child interactions, numerical board games are also well engineered to promote numerical understanding. The present study adds causal evidence to the previous correlational support for the view that playing numerical board games enhances the numerical understanding of children who play them.

Very recent research (Ramani & Siegler, 2008) indicates that the positive effects of playing numerical board games are not limited to improved number line estimation. Playing the game also improves the counting, number identification, and numerical magnitude comparison skills of preschoolers from low-income families. The gains endure for at least 9 weeks. The minimal cost and knowledge demands of playing the board games suggest that they could be widely adopted in preschool and Head Start centers, and that doing so would reduce the gap between lower- and middle-income children's early numerical understanding.

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